BRANE COSMOLOGY and Randall-Sundrum model

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  - Magnetic monopoles

One of the solutions

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When vaccum energy was the dominant component of the energy density of the universe,

$a(t)$ grew exponentially.

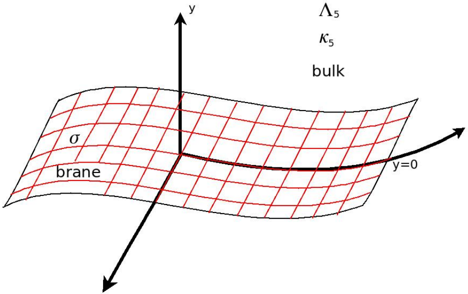
In a time interval $\Delta t \approx 10^{-32}[s]$

$a(t)$ grows by a factor of

$e^{100} \sim 3 \times 10^{43}$!

Brane Cosmology

One alternative cosmological model is Brane Cosmology. Embedding our typical 4D space-time in a higher 5D "space", and proposing that matter is confined to the brane, yields a model of universe consistent with requirements of General Relativity. Furthermore, it is required a fine-tunning relation between parameters in the brane and the bulk.



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- BULK:
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  - The brane is located at $y = 0$.
- 5D spacetime
  - x_0 : time coordinate
  - x_i : space coordinates, $i = 1, 2, 3$
  - y : additional dimension such that $y = -y$ (Z_2 symmetry)

Action

From an Action, we can derive equations of motion.
Einstein-Hilbert Action

$$S_H = \int \sqrt{-g} R d^4x$$

$$S = \frac{1}{16\pi G} S_H + S_M$$

Using principle of least action, it is obtained the Einstein Field Equations,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Randall-Sundrum model

In the Randall-Sundrum model, total action is a sum of the contribution of the bulk (5D space-time) and the brane (4D space-time).

$$S = S_{EH} + S_{brane}$$

$$S_{EH} = -\int d^5x\sqrt{-g^{(5)}}\left(\frac{R}{2\kappa_5^2} + \Lambda_5\right)$$

$$S_{brane} = \int d^4x\sqrt{-g^{(4)}}(-\sigma)$$

Solution of RS model

But here things are more complicated, it is not enough to calculate $\delta S = 0$...
The brane induces a discontinuity in the bulk, which is solved by imposing junction conditions.

$$G_{\mu\nu} \simeq K`_{\mu\nu} - K`\eta_{\mu\nu}$$

with $G_{\mu\nu}$ the Einstein tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$

$K_{\mu\nu}$ is the *extrinsic curvature* 4-tensor of the brane.

Solution of RS model

Using these, and proposing the previous ansatz for the metric,

$$ds^2 = e^{-2K(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

yields two equations for the parameters of the bulk and the brane,

$$6K'^2 = -\kappa_5^2\Lambda_5$$

$$3K'' = \kappa_5^2\sigma\delta(y)$$

solving both, we obtain $K(y) = \sqrt{\left(\frac{-\kappa_5^2}{6}\Lambda_5\right)}y$, and $6K'(y)|_{y=0} = \kappa_5^2\sigma$

Solution of RS model

Together, the two equations imply

$$\Lambda_5 = \frac{-\kappa_5^2}{6}\sigma^2$$

For static solutions to exist, a fine-tunning must exist between the brane tension and the bulk cosmological constant.
Setting $\kappa_5 \equiv 1$, and writing the bulk metric of the form

$$ds^2 = a^2 b^2 (dt^2 - dy^2) - a^2 \delta_{ij} dx^i dx^j$$

Leads us to

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

Solution of RS model

It is defined

$d\tau = abdt$, $aH = da/d\tau$, $a = e^{\alpha(t)}$, $aH = da/d\tau$

Playing with the equations, we obtain:

$$H^2 = \frac{\rho^2}{36} + \frac{\Lambda_5}{6} + \frac{\mu}{a^4}$$

with μ an integration constant. Splitting energy-density and pressure:

$\rho = \rho_M + \sigma$, $p = p_M + \sigma$ we obtain Friedmann's equation

$$H^2 = \frac{8\pi G}{3}\rho_M\left(1 + \frac{\rho_M}{2\sigma}\right) + \frac{\Lambda_4}{3} + \frac{\mu}{a^4}$$

Physical considerations

Identifying

$$\frac{8\pi G}{3} = \frac{\sigma}{18}$$

$$\frac{\Lambda_4}{3} = \frac{\sigma^2}{36} + \frac{\Lambda_5}{6}$$

and comparing with the fine-tunning relation $\Lambda_5 = \frac{-\kappa_5^2}{6}\sigma^2$, we see that $\Lambda_4 = 0$. If this is not satisfied, $\Lambda_4 \neq 0$.
From Friedmann's equation, if $\rho_M \gg \sigma$, $H \propto \rho_M$, instead of $H \propto \sqrt{\rho_M}$. It is obtained an expansion rate larger than in the standard model.

Physical considerations

Another models, more complicated, include a scalar field ϕ confined to the brane, obtaining inflation in the brane.
An interesting possibility, given by more sophisticated models, is a variation in the traditional potencial energy between two masses (confined on the brane):

$$V(r) = \frac{G^{(5)} m_1 m_2}{r} \left(1 + \frac{l^2}{r^2} + O(r^{-3})\right)$$

with l related to the parameters of the bulk:

$$l = \frac{-6}{\kappa_5^2 \Lambda_5}$$

REFERENCES

- D. Langlois, Prog.Theor.Phys.Suppl.148:181-212,2003
- Bruck et al 2003, arXiv:hep-th/0303095v1
- Papantonopoulos, Lect.Notes Phys. 592 (2002) 458
- Randall L, Sundrum R. 1999 Phys.Rev.Lett. 83 3370
- Randall L, Sundrum R. 1999 Phys.Rev.Lett. 83 4690
- Israel W 1966 Nuovo Cim B44S10 1; Erratum: ibid Nuovo Cim B48 463
- Battye, Carter, Phys.Lett. B, Volume 509, Issue 3-4, p. 331-336
- Kolb and Turner, *The Early Universe*
- Sean Carroll, *Spacetime and Geometry*.
- Weinberg, Gravitation and Cosmology.
- Peacock, Cosmological Physics.
- Misner, Thorne and Wheeler, *Gravitation*.

Thanks :)