VISIBILITY OF INTERFERENCE-FRINGES IN THE FOCUS OF A TELESCOPE.*

BY ALBERT A. MICHELSON.

When the angle subtended by an object viewed through a telescope is less than that subtended by a light-wave at a distance equal to the diameter of the objective, the form of the object can no longer be inferred from that of the image. Thus, if the object be a disk, a triangle, a point, or a double star, the appearance in the telescope is nearly the same.

If, however. the objective is limited by a rectangular slit, or, better, by two such, equal and parallel, then, as has been shown in a former paper,† the visibility of the interference-fringes is, in general, a periodic function of the ratio of α, the angular magnitude of the source in the direction perpendicular to the length of the slits, and α_0, the "limit of resolution." The period of this function, and thence $\frac{\alpha}{\alpha_0}$, may be found with great accuracy; so that by annulling the greater portion of the objective the accuracy of measurement of the angular magnitude of a small or distant source may be increased from ten to fifty times. As ordinarily understood, this increase of "accuracy" would be at the cost of "definition" (which, in this sense, is practically zero); but if by "definition" we mean, not the closeness of the resemblance of the image to the object, but the accurracy with which the form may be inferred, then definition and accuracy are increased in about the same proportion.

In almost every case likely to arise in practice, the form of the source is a circular disk; and if the illumination over its surface were uniform, the only problem to be solved would be the measurement of its diameter. But in many cases the distribution is anything but uniform. If the curve representing the distribution along the radius be $i = \psi(r)$, then the element of intensity of a strip $y_1 dx$ will be

$$\int_{-y_1}^{y_1} \psi(r)\, dy = \phi(x),$$

* Reprinted, by request, from the *Philosophical Magazine.*

† "On the Application of Interference Methods to Astronomical Measurements" (Phil. Mag., July, 1890).

and it has been shown that the visibility-curve in this case is

$$V = \frac{\int \phi(x) \cos kx}{\int \phi(x)\, dx}.$$

This may be proved as follows:

The intensity of the diffraction-figure of a luminous point in a telescope with a symmetrical aperture is*

$$I^2 = \left[\int\int \cos \kappa\mu_1 x_1 \cos \kappa\nu_1 y_1 dx_1 dy_1\right]^2, \quad \ldots \ldots \quad (1)$$

in which $k = 2\pi/\lambda$, μ_1 and v_1 are the angular distances from the center of the image, and x_1 and y_1 are the co-ordinates of the element of surface of the aperture.

If μ and ν are counted from the axis of the telescope and x, y, r are the co-ordinates of the luminous point, the expression becomes

$$I^2 = \left[\int\int \cos \kappa\left(\mu - \frac{x}{r}\right)x_1 \cos \kappa\left(\nu - \frac{y}{r}\right)y_1 dx_1 dy_1\right]^2. \quad . \quad (2)$$

If now the source is a luminous surface whose elements vibrate independently,

$$\bar{I} = \int\int I^2 dx\, dy. \quad \ldots \ldots \ldots \quad (3)$$

For the case of two equal apertures† whose centers are at $x_1 = -\frac{1}{2}a_{11}$ and $x_1 = +\frac{1}{2}a_{11}$,

$$I^2_{11} = I^2 \cos^2 \tfrac{1}{2} \kappa a_{11} \left(\mu - \frac{x}{r}\right). \quad \ldots \ldots \quad (4)$$

This substituted in (3) gives

$$\bar{I} = \int\int I^2 \cos^2 \tfrac{1}{2} \kappa a_{11} \left(\mu - \frac{x}{r}\right) dx\, dy.$$

Putting $\kappa a_{11}\mu = \theta$, $\kappa a_{11}/r = k_{11}$, and expanding,

$$2\bar{I} = \int\int I^2 dx\, dy + \cos\theta \int\int I^2 \cos k_{11}x\, dx\, dy$$
$$+ \sin\theta \int\int I^2 \sin k_{11}x\, dx\, dy. \quad \ldots \quad (5)$$

Let $y = \phi(x)$ be the equation of the curve bounding the luminous surface; or, better, let $\phi(x)dx$ be the "total intensity" of a strip of width dx.

Denoting $\int_{-\phi(x)}^{+\phi(x)} I^2 dy$ by $F(x)$, and omitting the factor 2,

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* "Wave Theory of Light," Rayleigh.

† More generally, for m equal equidistant apertures whose total area is constant,

$$I_{\mu} = I \frac{\sin \frac{1}{2} m\kappa\mu a}{m \sin \frac{1}{2} \kappa\mu a}.$$

equation (5) becomes

$$\bar{I}=\int F(x)dx+\cos\theta\int F(x)\cos kx\,dx+\sin\theta\int F(x)\sin kx\,dx,$$

or

$$\bar{I}=P+C\cos\theta+S\sin\theta. \qquad (6)$$

If the width of the apertures is small, compared with their distance, the variations of $F(x)$ with μ (or θ) may be neglected, and in this case the maxima or minima occur when

$$\tan\theta=\frac{S}{C},\ \text{or when}\ \bar{I}=P\pm\sqrt{C^2+S^2}.$$

If now the visibility of the interference-fringes be defined as the ratio of the difference between a maximum and an adjacent minimum to their sum

$$V^2=\frac{C^2+S^2}{P^2},$$

or

$$V^2=\frac{[\int F(x)\cos kx\,dx]^2+[\int F(x)\sin kx\,dx]^2}{[\int F(x)\,dx]^2}. \qquad (7)$$

For narrow rectangular apertures,

$$F(x)=\int_{w_1}^{w_2}\frac{\sin^2 w}{w}dw.$$

In this expression, if v=o and b=length of aperture,

$$w_1=\frac{\kappa b}{2r}\phi_1(x)\ \text{and}\ w_2=\frac{\kappa b}{2r}\phi_2(x).$$

So long as

$$\frac{2\phi(x)}{r}<\frac{\lambda}{b},$$

$F(x)$ is nearly proportional to $\phi(x)$; that is, so long as the angle subtended by the source is less than the limit of resolution of a telescope with aperture b, the brightness is proportional to the size of the object. For larger angles the proportionality may still be made to hold by a slight alteration in the focal adjustment; and to this degree of approximation we have

$$V_2=\frac{[\int\phi(x)\cos kx\,dx]^2+[\int\phi(x)\sin kx\,dx]^2}{[\int\phi(x)\,dx]^2}. \qquad (8)$$

If the source is symmetrical the second term vanishes, and the expression reduces to the original form.

It is possible that, in addition to the uses already mentioned, the "visibility-curve" may have an important application in the case of small spherical nebulæ. For from the form of this curve the distribution of luminous intensity in the globular mass may be inferred, which would furnish a valuable clue to the distribution of temperature and density in gaseous nebulæ.

When the source is so small as to be indistinguishable from a star, it would seem that this method is the only one capable of giving reliable information; but even in the case of bodies of larger apparent size it is equally applicable, may be made to give results at least as accurate as could be obtained by photometric measurements, and is far more readily applied.

REPORT MADE TO THE DIRECTOR OF THE ASTRONOMICAL OBSERVATORY OF TACUBAYA, IN REGARD TO OBSERVATIONS OF THE ZODIACAL LIGHT.*

The total eclipse of the Sun that took place on the 22d of December, 1889, presented exceptionally good conditions to study the Zodiacal light and crepuscular phenomena, on account of the fact that the zone of totality and its extension crossed our planet in the intertropical regions, where such phenomena take place with greater intensity and under better conditions for their observation; besides, the eclipse occurred at the time when the Zodiacal light shows its greatest extension and brightness. The eclipse began at sunrise for the occidental coast of America, and at sunset for the western coast of Africa. Therefore, the shadow of the Moon touched the Earth at the time when the Zodiacal light is seen distinctly, so that a rare opportunity was offered to the observers, to ascertain with certainty, whether or not the Zodiacal light is produced (at least in part) between the Earth and the Moon, or at a greater distance than that between our planet and its satellite. In order to observe the above-mentioned phenomena Sres. D. CAMILO A. GONZALEZ and D. FELIPE VALLE, of the Astronomical Observatory of Tacubaya, went to Progreso, Yucatan, Mexico.

The observations of the Zodiacal light extended from the 14th to the 25th of December, *i. e.*, seven days before and three days after the day of the eclipse, which took place on the 22d.

* Translated from the *Boletin del Observatorio Astronomico Nacional de Tacubaya*, by E. J. MOLERA.

There was no difference of any importance in the aspect of the Zodiacal light during the days preceding or following the day of the eclipse, but, "on the 22d, at $5^h\ 8^m\ 19^s$ A. M., common time at Progreso (the latitude of which is 21° 17′ 14″.3 N., and its longitude, from the Astronomical Observatory at Tacubaya, 38° 08′ E., or $5^h\ 58^m\ 38^s.2$ W. of Greenwich), at the moment we were watching with great attention the Zodiacal light, *we saw something like a veil or shadow spread itself over it, and diminish its intensity about one-half.* The phenomenon was noticed by the two observers *independently*, and the impressions that the phenomena made on both, *were identical.* It was due, without doubt, to the cone of shadow of the Moon projected on the matter that reflects the solar light directly, after it is reflected by our own planet. Accordingly, the phenomena took place several minutes before the totality of the eclipse began, as it should do, the matter that reflects the light, and produces the phenomenon being at a distance of many thousands of kilometers from the center of the Earth; the shadow, though lasting only a few moments, was gradual, and moved from the zenith to the horizon, or from west to east, in the exact direction that the march of the intersection of the cone of shadow of the Moon with the Earth followed. The notes of observation were:

December 22d, 1889, $3^h\ 55^m$ A. M.—The Zodiacal light is distinctly visible between α and β *Libræ* and δ and μ *Leonis*, and possibly as far as θ *Leonis.*

$4^h\ 49^m$.—It is seen more brilliantly in some places in the region that it occupies, for instance, near ζ *Libræ*, and it is considerable more brilliant than at the horizon.

$5^h\ 08^m\ 19^s$.—*The shadow of the Moon is projected over the Zodiacal light*, reducing the intensity of its brilliancy one-half.

$5^h\ 15^m$.—The most brilliant part of the Zodiacal light is at the extreme occidental side of *Scorpio.*

NOTE ON DARK TRANSITS OF *JUPITER'S* SATELLITES.

By John Tebbutt, F. R. A. S.

I have read with much interest the notices which have appeared in Nos. 10 and 11 of the *Publications* A. S. P. with reference to black transits of *Jupiter's* satellites. I have myself